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Theory:

Equation of motion of multi degree of freedom system in matrix form

$$[m]\{x''\} + [c]\{x'\} + [k]\{x\} = -x_g(t)[m]\{I\}$$

where,

$[m]$ = Mass Matrix.

$[k]$ = Stiffness Matrix.

$[c]$ = Damping Matrix

$\{I\}$ = Unit Vector

$x''_g(t)$ = Ground Motion

The physical coordinates $\{x\}$ may be related with normal or principal coordinates $\{q\}$ as,

$$\{x\} = [\varphi]\{q\}$$

where, $[\varphi]$ = Modal Matrix.

Time Derivate of $\{x\}$ are

$$\{x'\} = [\varphi]\{q'\}$$

$$\{x''\} = [\varphi]\{q''\}$$

Substituting the time derivatives in equation of motion and pre multiplying by φ^T results.

$$[\varphi]^T [m] [\varphi] \{q''\} + [\varphi]^T [c] [\varphi] \{q'\} + [\varphi]^T [k] [\varphi] \{q\} = (-x''_g(t) [\varphi]^T [m] \{I\})$$
$$[m]\{q''\} + [c]\{q'\} + [k]\{q\} = \{P_{\text{eff}}(t)\}$$

Modal Orthogonality:

$$[M] = [\varphi]^T [m] [\varphi]$$

$$[C] = [\varphi]^T [c] [\varphi]$$

$$[K] = [\varphi]^T [k] [\varphi]$$

$$\{P_{\text{eff}}(t)\} = (-x''_g(t) [\varphi]^T [m] \{I\})$$

$[m]$, $[c]$, and $[K]$ are the diagonalized modal mass matrix, modal damping matrix and stiffness matrix and $P_{\text{eff}}(t)$ is the effective modal force vector.

Finally, by using numerical methods like Newark's linear acceleration method, Newark's average acceleration method, central difference method etc. We will find the response of each floor of given structure.