



Theory –

1. Bus Type –

Bus Type	Explanation	Known	Unknown
Load bus or PQ bus	A bus connected only to loads, i.e., P and Q, are known as a load bus. Generally, P and Q are specified for such type of buses.	P and Q	V and δ
Generator bus or PV bus	A bus connected to a generator are known as a generator bus. In general, P and V of such buses are known. A generator can maintain the voltage on a bus till its reactive power capability which is important for this bus to continue to operate as a PV bus.	P and V	Q and δ
Slack or reference bus	One bus in a system is specified as a slack bus whose V and δ are specified and the quantities P and Q are calculated. Here, please note that P is unknown for this bus which takes care of the mismatch in the generation and losses. Generally, the largest generator in the system is considered as the slack bus.	V and δ	P and Q

2. Power Flow Equations –

$$\begin{bmatrix} I_1 \\ I_2 \\ \vdots \\ I_i \\ \vdots \\ I_n \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} & \dots & Y_{1i} & \dots & Y_{1n} \\ Y_{21} & Y_{22} & \dots & Y_{2i} & \dots & Y_{2n} \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ Y_{i1} & Y_{i2} & \dots & Y_{ii} & \dots & Y_{in} \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ Y_{n1} & Y_{n2} & \dots & Y_{ni} & \dots & Y_{nn} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ \vdots \\ V_i \\ \vdots \\ V_n \end{bmatrix} \quad (1)$$

$$I_{bus} = Y_{bus} V_{bus}$$

Where, I_{bus} is the vector of the injected bus currents and V_{bus} is the vector of bus voltages measured from the reference node. Y_{bus} is known as the bus admittance matrix.

For each bus i :

$$I_i = \sum_{j=1}^n Y_{ij} V_j$$

$$I_i = \sum_{j=1}^n Y_{ij} V_j = Y_{ii} V_i + \sum_{j=1, j \neq i}^n Y_{ij} V_j \quad (2)$$

Complex power injection at bus i is given by, $S_i = P_i + jQ_i = V_i I_i^*$. Inserting (2) and separating real and imaginary terms, the power flow equations for active power and reactive power are obtained and given in (3) and (4).

$$P_i = \sum_{j=1}^n |V_i| |V_j| |Y_{ij}| \cos(\theta_{ij} + \delta_j - \delta_i) \quad (3)$$

$$Q_i = - \sum_{j=1}^n |V_i| |V_j| |Y_{ij}| \sin(\theta_{ij} + \delta_j - \delta_i) \quad (4)$$

Where, $|V_i|$ and δ_i are the voltage magnitude and angle at bus i , and $|Y_{ij}|$ and θ_{ij} are the magnitude and angle of the admittance matrix corresponding to the element at i^{th} row and j^{th} column. For a 'n' bus system, there are total '2n' load flow equations and '2n' variables.

Newton-Raphson method for power flow analysis –

The Newton-Raphson method is an iterative technique used to solve nonlinear algebraic equations. The Newton-Raphson method is based on the Taylor series expansion and can handle large power systems effectively for load flow analysis. The method involves calculating the Jacobian matrix, power mismatch vector, and applying corrections to the bus voltages iteratively until the minimization of the power mismatches. The advantages of the Newton-Raphson method are listed below –

- The quadratic convergence in the Newton-Raphson method makes it faster than the Gauss-Seidel method.
- Due to the faster convergence of the Newton-Raphson method, it solves large and complex power systems effectively.

Here's the general procedure for a Newton-Raphson method:

1. **Formulation of power flow equations:** The power flow equations are nonlinear equations relating bus voltages the power injections at each bus. For a bus i , the real and reactive power injections are given by -

$$P_i = \sum_{j=1}^N |V_i| |V_j| |Y_{ij}| \cos(\theta_{ij} + \delta_j - \delta_i) \quad (3)$$

$$Q_i = - \sum_{j=1}^N |V_i| |V_j| |Y_{ij}| \sin(\theta_{ij} + \delta_j - \delta_i) \quad (4)$$

2. **Initial guess:** For a N-bus power system, assume bus 1 as the slack bus, bus 2 to bus M are the PV buses, and bus M+1 to N are the PQ buses. An initial guess is made for the voltage magnitudes and angles for all buses except the slack bus. For PV buses, voltage magnitude is fixed, and angle is guessed. The unknown quantities are 2N-M-1.
3. **Mismatch vector:** Applying Taylor series expansion (till first order) for the real and reactive power flow equations –

$$P_i^{spec} = P_i(\delta_i^{(0)}, V_i^{(0)}) + \frac{\partial P_i}{\partial \delta_2} \Delta \delta_2 + \frac{\partial P_i}{\partial \delta_3} \Delta \delta_3 + \dots + \frac{\partial P_i}{\partial \delta_N} \Delta \delta_N + \frac{\partial P_i}{\partial V_{M+1}} \Delta V_{M+1} + \frac{\partial P_i}{\partial V_{M+2}} \Delta V_{M+2} \dots + \frac{\partial P_i}{\partial V_N} \Delta V_N, \quad \forall i = 2, \dots, N. \quad (5)$$

$$Q_i^{spec} = Q_i(\delta_i^{(0)}, V_i^{(0)}) + \frac{\partial Q_i}{\partial \delta_2} \Delta \delta_2 + \frac{\partial Q_i}{\partial \delta_3} \Delta \delta_3 + \dots + \frac{\partial Q_i}{\partial \delta_N} \Delta \delta_N + \frac{\partial Q_i}{\partial V_{M+1}} \Delta V_{M+1} + \frac{\partial Q_i}{\partial V_{M+2}} \Delta V_{M+2} \dots + \frac{\partial Q_i}{\partial V_N} \Delta V_N, \quad \forall i = 2, \dots, N. \quad (6)$$

Rearranging the terms and writing them into matrix form, then -

$$\begin{bmatrix} \frac{\partial P_2}{\partial \delta_2} & \frac{\partial P_2}{\partial \delta_3} & \cdots & \frac{\partial P_2}{\partial \delta_N} & \frac{\partial P_2}{\partial V_{M+1}} & \frac{\partial P_2}{\partial V_{M+2}} & \cdots & \frac{\partial P_2}{\partial V_N} \\ \frac{\partial P_3}{\partial \delta_2} & \frac{\partial P_3}{\partial \delta_3} & \cdots & \frac{\partial P_3}{\partial \delta_N} & \frac{\partial P_3}{\partial V_{M+1}} & \frac{\partial P_3}{\partial V_{M+2}} & \cdots & \frac{\partial P_3}{\partial V_N} \\ \vdots & \vdots & & \vdots & \vdots & \vdots & & \vdots \\ \frac{\partial P_N}{\partial \delta_2} & \frac{\partial P_N}{\partial \delta_3} & \cdots & \frac{\partial P_N}{\partial \delta_N} & \frac{\partial P_N}{\partial V_{M+1}} & \frac{\partial P_N}{\partial V_{M+2}} & \cdots & \frac{\partial P_N}{\partial V_N} \\ \frac{\partial Q_{M+1}}{\partial \delta_2} & \frac{\partial Q_{M+1}}{\partial \delta_3} & \cdots & \frac{\partial Q_{M+1}}{\partial \delta_N} & \frac{\partial Q_{M+1}}{\partial V_{M+1}} & \frac{\partial Q_{M+1}}{\partial V_{M+2}} & \cdots & \frac{\partial Q_{M+1}}{\partial V_N} \\ \vdots & \vdots & & \vdots & \vdots & \vdots & & \vdots \\ \frac{\partial Q_N}{\partial \delta_2} & \frac{\partial Q_N}{\partial \delta_3} & \cdots & \frac{\partial Q_N}{\partial \delta_N} & \frac{\partial Q_N}{\partial V_{M+1}} & \frac{\partial Q_N}{\partial V_{M+2}} & \cdots & \frac{\partial Q_N}{\partial V_N} \end{bmatrix} \begin{bmatrix} \Delta \delta_2 \\ \Delta \delta_3 \\ \vdots \\ \Delta \delta_N \\ \Delta V_{M+1} \\ \Delta V_{M+2} \\ \vdots \\ \Delta V_N \end{bmatrix} = \begin{bmatrix} P_2^{spec} - P_2^{cal} \\ P_3^{spec} - P_3^{cal} \\ \vdots \\ P_N^{spec} - P_N^{cal} \\ Q_{M+1}^{spec} - Q_{M+1}^{cal} \\ Q_{M+2}^{spec} - Q_{M+2}^{cal} \\ \vdots \\ Q_N^{spec} - Q_N^{cal} \end{bmatrix} \quad (7)$$

Equation (7) is represented by –

$$\begin{bmatrix} J_1 & J_2 \\ J_3 & J_4 \end{bmatrix} \begin{bmatrix} \Delta \delta \\ \Delta V \end{bmatrix} = \begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix} \quad (8)$$

Where,

$$[\delta] = [\delta_2 \ \delta_3 \ \cdots \ \delta_N], \quad [V] = [V_{M+1} \ V_{M+2} \ \cdots \ V_N], \quad [P] = [P_2 \ P_3 \ \cdots \ P_N], \quad \text{and} \quad [Q] = [Q_{M+1} \ Q_{M+2} \ \cdots \ Q_N].$$

$$[\Delta \delta] = [\Delta \delta_2 \ \Delta \delta_3 \ \cdots \ \Delta \delta_N]_{(N-1) \times 1}^T, \quad [\Delta V] = [\Delta V_{M+1} \ \Delta V_{M+2} \ \cdots \ \Delta V_N]_{(N-M) \times 1}^T,$$

$$[\Delta P] = [\Delta P_2 \ \Delta P_3 \ \cdots \ \Delta P_N]_{(N-1) \times 1}^T, \quad \text{and} \quad [\Delta Q] = [\Delta Q_{M+1} \ \Delta Q_{M+2} \ \cdots \ \Delta Q_N]_{(N-M) \times 1}^T.$$

$$\text{Where, } [J_1] = \left[\frac{\partial P}{\partial \delta} \right]_{(N-1) \times (N-1)}, \quad [J_2] = \left[\frac{\partial P}{\partial V} \right]_{(N-1) \times (N-M)}, \quad [J_3] = \left[\frac{\partial Q}{\partial \delta} \right]_{(N-M) \times (N-1)}, \quad \text{and} \quad [J_4] = \left[\frac{\partial Q}{\partial V} \right]_{(N-M) \times (N-M)}.$$

Equation (8) is further simplified as given below –

$$[J][\Delta X] = [mis] \quad (9)$$

Where, $\Delta X = \begin{bmatrix} \Delta \delta \\ \Delta V \end{bmatrix}$ is the correction vector and $mis = \begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix}$ is the real and reactive power mismatch vector. ΔP and ΔQ for each bus are calculated (except slack bus). The power mismatch vector for real and reactive power is given below, where P^{spec}/Q^{spec} are the scheduled active and reactive power and P^{cal}/Q^{cal} are the calculated active and reactive power at a bus i substituting the initial guess.

$$[mis] = \begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix} = \begin{bmatrix} P^{spec} - P^{cal} \\ Q^{spec} - Q^{cal} \end{bmatrix}$$

To perform the power flow analysis, an initial guess for the bus voltage magnitude is required. For the normal steady-state operating conditions, the bus voltage magnitudes maintain between 0.95 – 1.05 p.u. Therefore, all the unknown bus voltages are initialized at 1.0∠0° p.u., also called as a flat start. Let us assume that the first bus is a slack bus and from bus 2 to ‘M’ bus are the PV buses for a ‘N’ bus system. The remaining ‘N-M-1’ buses are the PQ buses. The complete procedure for Newton-Raphson power flow is as given below -

Step 1: Initialize $V_j^{(0)} = V_j^{(spec)} \angle 0^\circ$ for $j = 2, 3, \dots, m$ and $V_j^{(0)} = 1 \angle 0^\circ$ for $j = (m + 1), (m + 2), \dots, n$. Let the initial vectors for voltage magnitudes and angles be denoted as $V^{(0)}$ and $\delta^{(0)}$, respectively. Set iteration count $k=1$.

Step 2: Perform the following operations for $i = 2, 3, \dots, m$.

a) Calculate:

$$Q_i^{(k)} = - \sum_{j=1}^n |V_i^{(k-1)}| |V_j^{(k-1)}| |Y_{ij}| \sin(\theta_{ij} + \delta_j^{(k-1)} - \delta_i^{(k-1)}) \quad (10)$$

- b) If $Q_i^{min} < Q_i^{(k)} < Q_i^{max}$, then assign $|V_i^{(k)}| = V_i^{(spec)}$ and the i^{th} bus is retained as PV bus for k^{th} iteration.
- c) If $Q_i^{(k)} > Q_i^{max}$, then set $Q_i^{spec} = Q_i^{max}$, and if $Q_i^{(k)} < Q_i^{min}$, then set $Q_i^{spec} = Q_i^{min}$. In both scenarios, the respective bus is converted to PQ bus. The voltage of the respective bus becomes an unknown for the current iteration. It introduces extra unknown quantities and to evaluate this, an extra equation is needed, which is obtained from (10). Therefore, when the i^{th} bus is converted to the PQ bus, the dimensions of both ΔP and ΔQ are increased by one.

Step 3: Compute the vectors P^{cal} and Q^{cal} with the vectors $\delta^{(k-1)}$ and $V^{(k-1)}$ to form the mismatch vector ΔM .

Step 4: Compute $error = \max(|\Delta M|)$.

Step 5: If $error \leq \epsilon(pre - specified tolerance)$, then the final vectors are $\delta^{(k-1)}$ and $V^{(k-1)}$ and print the results. Otherwise go to step 6.

Step 6: Evaluate the Jacobian matrix with the vectors $\delta^{(k-1)}$ and $V^{(k-1)}$.

Step 7: Evaluate the correction vector ΔX by solving equation (9).

Step 8: Update the solution vectors $\delta^{(k)} = \delta^{(k-1)} + \Delta\delta$ and $V^{(k)} = V^{(k-1)} + \Delta V$. Update $k=k+1$ and go back to step 2.