



Theory:

The known and unknown quantities of these buses are listed below

1. Bus Type –

Bus Type	Explanation	Known	Unknown
Load bus or PQ bus	A bus connected only to loads, i.e., P and Q, are known as a load bus. Generally, P and Q are specified for such type of buses.	P and Q	V and δ
Generator bus or PV bus	A bus connected to a generator are known as a generator bus. In general, P and V of such buses are known. A generator can maintain the voltage on a bus till its reactive power capability which is important for this bus to continue to operate as a PV bus.	P and V	Q and δ
Slack or reference bus	One bus in a system is specified as a slack bus whose V and δ are specified and the quantities P and Q are calculated. Here, please note that P is unknown for this bus which takes care of the mismatch in the generation and losses. Generally, the largest generator in the system is considered as the slack bus.	V and δ	P and Q

2. Power Flow Equations –

$$\begin{bmatrix} I_1 \\ I_2 \\ \vdots \\ I_i \\ \vdots \\ I_n \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} & \dots & Y_{1i} & \dots & Y_{1n} \\ Y_{21} & Y_{22} & \dots & Y_{2i} & \dots & Y_{2n} \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ Y_{i1} & Y_{i2} & \dots & Y_{ii} & \dots & Y_{in} \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ Y_{n1} & Y_{n2} & \dots & Y_{ni} & \dots & Y_{nn} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ \vdots \\ V_i \\ \vdots \\ V_n \end{bmatrix} \quad (1)$$

$$I_{bus} = Y_{bus} V_{bus}$$

Where, I_{bus} is the vector of the injected bus currents and V_{bus} is the vector of bus voltages measured from the reference node. Y_{bus} is known as the bus admittance matrix.

For each bus i :

$$I_i = \sum_{j=1}^n Y_{ij} V_j$$

$$I_i = \sum_{j=1}^n Y_{ij} V_j = Y_{ii} V_i + \sum_{j=1, j \neq i}^n Y_{ij} V_j \quad (2)$$

Complex power injection at bus i is given by, $S_i = P_i + jQ_i = V_i I_i^*$. Inserting (2) and separating real and imaginary terms, the power flow equations for active power and reactive power are obtained and given in (3) and (4).

$$P_i = \sum_{j=1}^n |V_i| |V_j| |Y_{ij}| \cos(\theta_{ij} + \delta_j - \delta_i) \quad (3)$$

$$Q_i = - \sum_{j=1}^n |V_i| |V_j| |Y_{ij}| \sin(\theta_{ij} + \delta_j - \delta_i) \quad (4)$$

Where, $|V_i|$ and δ_i are the voltage magnitude and angle at bus i , and $|Y_{ij}|$ and θ_{ij} are the magnitude and angle of the admittance matrix corresponding to the element at i^{th} row and j^{th} column. For a 'n' bus system, there are total '2n' load flow equations and '2n' variables.

Gauss-Seidel method for power flow analysis –

The Gauss-Seidel method is an iterative technique used to solve nonlinear algebraic equations. The Gauss-Seidel method iteratively updates the voltage at each bus using the latest updated values from the previous steps. Here's the general procedure for a Gauss-Seidel method:

$$I_i = \sum_{j=1}^n Y_{ij} V_j = Y_{ii} V_i + \sum_{j=1, j \neq i}^n Y_{ij} V_j$$

By rearranging the above equation, we get,

$$V_i = \frac{1}{Y_{ii}} \left(I_i - \sum_{j=1, j \neq i}^n Y_{ij} V_j \right)$$

Also, from $S_i = P_i + jQ_i = V_i I_i^*$, I_i in the above equation can be replaced by $I_i = \frac{P_i - jQ_i}{V_i^*}$.

Rewriting the above equation by replacing I_i as,

$$V_i = \frac{1}{Y_{ii}} \left(\frac{P_i - jQ_i}{V_i^*} - \sum_{j=1, j \neq i}^n Y_{ij} V_j \right)$$

To perform the power flow analysis, an initial guess for the bus voltage magnitude is required. For the normal steady-state operating conditions, the bus voltage magnitudes maintain between 0.95 – 1.05 p.u. Therefore, all the unknown bus voltages are initialized at $1.0 \angle 0^\circ$ p.u., also called as a flat start. Let us assume that the first bus is a slack bus and from bus 2 to 'm' bus are the PV buses for a 'n' bus system. The remaining 'n-m-1' buses are the PQ buses. The complete procedure for Gauss-Seidel power flow is as given below -

Step 1: Initialize $V_j^{(0)} = V_j^{(spec)} \angle 0^\circ$ for $j = 2, 3, \dots, m$ and $V_j^{(0)} = 1 \angle 0^\circ$ for $j = (m+1), (m+2), \dots, n$. Set iteration count $K=1$.

Step 2: Perform the following operations for $i = 2, 3, \dots, m$.

a) Calculate:

$$Q_i^{(k)} = - \sum_{j=1}^n |V_i^{(k-1)}| |V_j^{(k-1)}| |Y_{ij}| \sin(\theta_{ij} + \delta_j^{(k-1)} - \delta_i^{(k-1)})$$

b) If $Q_i^{min} < Q_i^{(k)} < Q_i^{max}$, then assign $|V_i^{(k)}| = V_i^{(spec)}$ and $\delta_i^{(k)} = \angle(A_i^{(k)})$ where $A_i^{(k)}$ is given by,

$$A_i^{(k)} = \frac{1}{Y_{ii}} \left(\frac{P_i - jQ_i^{(k)}}{\{V_i^{(k-1)}\}^*} - \sum_{j=1}^{i-1} Y_{ij} V_j^{(k)} - \sum_{j=i+1}^n Y_{ij} V_j^{(k-1)} \right)$$

c) If $Q_i^{(k)} \geq Q_i^{max}$, then calculate,

$$V_i^{(k)} = \frac{1}{Y_{ii}} \left(\frac{P_i - jQ_i^{max}}{\{V_i^{(k-1)}\}^*} - \sum_{j=1}^{i-1} Y_{ij} V_j^{(k)} - \sum_{j=i+1}^n Y_{ij} V_j^{(k-1)} \right)$$

d) If $Q_i^{(k)} \leq Q_i^{min}$, then calculate,

$$V_i^{(k)} = \frac{1}{Y_{ii}} \left(\frac{P_i - jQ_i^{min}}{\{V_i^{(k-1)}\}^*} - \sum_{j=1}^{i-1} Y_{ij} V_j^{(k)} - \sum_{j=i+1}^n Y_{ij} V_j^{(k-1)} \right)$$

Step 3: For $i = (m + 1) \dots n$, calculate,

$$V_i^{(k)} = \frac{1}{Y_{ii}} \left(\frac{P_i - jQ_i^{(k)}}{\{V_i^{(k-1)}\}^*} - \sum_{j=1}^{i-1} Y_{ij} V_j^{(k)} - \sum_{j=i+1}^n Y_{ij} V_j^{(k-1)} \right)$$

Step 4: Compute $e_i^{(k)} = |V_i^{(k)} - V_i^{(k-1)}|$ for all $i = 2, 3, \dots, n$;

Step 5: Compute $e^{(k)} = \max(e_2^{(k)}, e_3^{(k)}, \dots, e_n^{(k)})$;

Step 6: If $e^{(k)} < \epsilon$, stop and print the solution. Else set $k = k + 1$, and go to step 2.