Earthquake Engineering Research Centre
International Institute of Information Technology
Gachibowli, Hyderabad – 500 032, India

Theory:

Free vibration is initiated by disturbing the system from its static equilibrium position by imparting the mass some displacement u(0) and velocity u'(0) at time t=0.

There are two cases in free vibration
1. undamped free vibration
2. damped free vibration

Undamped free vibration :
The governing equation for undamped free vibration is

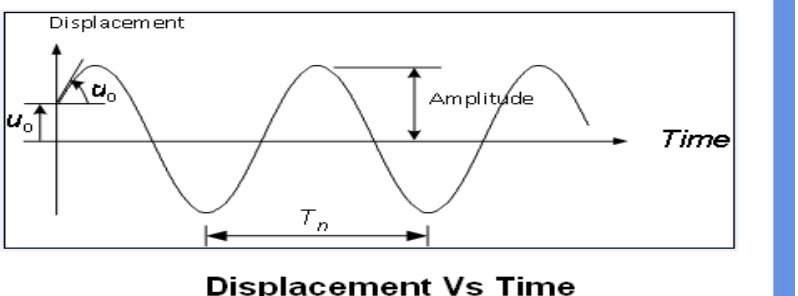
$$m\ddot{u} + ku = 0$$

where,
m = mass ,u'' = acceleration, k = stiffness, u = displacement.
The solution to homogeneous equation is :

$$u(t) = u(0)\cos \omega_n t + \frac{\dot{u}(0)}{\omega_n}\sin\ \omega_n t$$

where,
u(0) = initial displacement ,u'(0) = initial velocity.

Here in this solution we can observe that the system will vibrate only if initial displacement and/or initial velocity is given.



Displacement Vs Time

Damped free vibration :

The governing free vibration of the SDF system with damping

$$m\ddot{u} + c\dot{u} + ku = 0$$

where,
c = damping coefficient , u' = velocity coefficient

$$\ddot{u} + 2\varsigma\omega_n\dot{u} + \omega_n^2 u = 0$$

where ω_n = natural frequency